Extracting Jump Intensity From The DDM

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In this white paper we will extract jump intensity implied by the dividend discount model where the discount rate is adjusted to account for jump risk rather than modeling jump risk explicitly. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

ABC Company's revenue and cash flow are dependent on a one major customer relationship. ABC Company's current revenue mix is...

Table 1: Revenue Mix

Customer	Percent
Largest customer	40
Second largest customer	30
Third largest customer	20
All other customers	10
Total	100

We are tasked with building a model to estimate (1) the probability that all three major customer relationships would survive the next three years and (2) the weighted-average remaining life of eash customer relationship. We are to assume that if there is a jump (i.e. a customer relationship ends) then cash flow will decline by the same percentage as total revenue. Our task is to answer the following questions...

Question 1: What is the jump intensity implied by the dividend discount model given that the discount rate is increased by 300 bps to compensate for jump risk?

Question 2: What is expected customer relationship life in years?

Question 3: What is probability that all three major customer relationships would survive the next three years?

Jump Diffiusion Model For Company Value

We will define the variable C_t to be annualized cash flow at time t, the variable μ to be the continuous-time cash flow growth rate, the variable σ to be the cash flow growth rate volatility, the variable ω to be jump size, the variable κ to be the continuous-time risk-adjusted discount rate before jump risk, the variable z to be a normally-distributed random variate, the variable n to be the Poisson-distributed random number of jumps, and the variable λ to be jump intensity. The jump diffusion equation for random annualized cash flow at time t is... [4]

$$C_t = C_0 \,\mathrm{Exp}\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\,z + n\,\ln\left(1+\omega\right)\right\} \text{ ...where... } z \sim N\left[0,1\right] \text{ ...and... } n \sim P\left[\lambda\,t, \lambda\,t\right] \tag{1}$$

The equation for the probability of drawing a random variable of value z from a standardized normal distribution is... [2]

$$\mathrm{Prob}\left[z\right] = \frac{1}{\sqrt{2\,\pi}}\,\mathrm{Exp}\left\{-\frac{1}{2}\,z^2\right\} \tag{2}$$

The equation for the probability of drawing a random variable of value n from a Poisson distribution is... [3]

$$\mathrm{Prob}\left[n\right] = \frac{(\lambda\,t)^n}{n!}\,\mathrm{Exp}\left\{-\lambda\,t\right\} \tag{3}$$

Using Equations (1), (2) and (3) above the equation for expected annualized cash flow at time t is... [4]

$$\mathbb{E}\bigg[C_t\bigg] = \mathbb{E}\bigg[C_0 \,\mathrm{Exp}\bigg\{\bigg(\mu - \frac{1}{2}\,\sigma^2\bigg)\,t + \sigma\sqrt{t}\,z + n\,\ln\bigg(1+\omega\bigg)\bigg\}\bigg] = C_0 \,\mathrm{Exp}\bigg\{\mu\,t\bigg\}\,\mathrm{Exp}\bigg\{\lambda\,\omega\,t\bigg\} \tag{4}$$

We will define the variable V_0 to be company value at time zero and the variable κ to be the risk-adjusted discount rate. The equation for company value at time zero is...

$$V_0 = \int_0^\infty \mathbb{E}\bigg[C_t\bigg]\,\mathrm{Exp}\bigg\{-\kappa\,t\bigg\} \tag{5}$$

Using Equation (4) above the solution to Equation (5) above is...

$$V_0 = \int_0^\infty C_0\,\mathrm{Exp}\bigg\{\mu\,t\bigg\}\,\mathrm{Exp}\bigg\{\lambda\,\omega\,t\bigg\}\,\mathrm{Exp}\bigg\{-\kappa\,t\bigg\} = \frac{C_0}{\kappa-\mu-\lambda\,\omega} \text{ ...when... } \kappa > \mu + \lambda\,\omega \tag{6}$$

Standard Dividend Discount Model For Company Value

If we remove the jump component from expected annualized cash flow Equation (4) above then that equation becomes...

$$\mathbb{E}\bigg[C_t\bigg] = C_0\,\mathrm{Exp}\bigg\{\mu\,t\bigg\} \tag{7}$$

Rather than model jumps directly we will remove the jump compent and compensate for that removal by increasing the discount rate by the variable ϕ such that our new discount rate is $\kappa + \phi$. Using Equations (6) and (7) above the equation for company value at time zero is...

$$V_0 = \int_0^\infty \mathbb{E}\bigg[C_t\bigg]\,\mathrm{Exp}\bigg\{-(\kappa+\phi)\,t\bigg\} = \int_0^\infty C_0\,\mathrm{Exp}\bigg\{\mu\,t\bigg\}\,\mathrm{Exp}\bigg\{-(\kappa+\phi)\,t\bigg\} = \frac{C_0}{\kappa+\phi-\mu} \tag{8}$$

Solving For DDM Implied Jump Intensity

To solve for jump intensity we will equate Equations (6) and (8) above and solve for jump intensity (λ). The equation for jump intensity is...

$$\text{if... } \frac{C_0}{\kappa-\mu-\lambda\,\omega} = \frac{C_0}{\kappa+\phi-\mu} \text{ ...then... } \lambda = -\frac{\phi}{\omega} \tag{9}$$

Using Equation (9) above the equation for jump probability is...

$$\text{Jump probability} = \text{Jump intensity} = \lambda \tag{10}$$

Using Equation (9) above the equation for weighted-average customer relationship life in years is... [1]

$$\text{Weighted-average customer life} = \text{WAL} = \frac{1}{\lambda}\text{ years} \tag{11}$$

The Answers To Our Hypothetical Problem

Using Table 1 above the equation for jump size (ω) is...

$$\omega = \frac{0.40+0.30+0.20}{3} = 0.30 \tag{12}$$

Per the hypothetical problem above the value of the variable ϕ (discount rate markup to compensate for jump risk) is...

$$\phi = 0.0300 \tag{13}$$

Question 1: What is the jump intensity implied by the dividend discount model given that the discount rate is increased by 300 bps to compensate for jump risk?

Using Equations (9), (12) and (13) above the answer to the question is...

$$\lambda = -\frac{\phi}{\omega} = -\frac{0.0300}{-0.3000} = 0.1000 \tag{14}$$

Question 2: What is expected customer relationship life in years?

Using Equations (11) and (14) above the answer to the question is...

$$\lambda = \frac{1}{\lambda} = \frac{1}{0.1000} = 10\,\text{years} \tag{15}$$

Question 3: What is probability that all three major customer relationships would survive the next three years (i.e. there are no jumps)?

Using Equations (3) and (14) above the answer to the question is...

$$\text{Prob}\left[n = 0\right] = \frac{(0.10 \times 3)^0}{0!} \times \text{Exp}\left\{ -0.10 \times 3 \right\} = 0.7408 \tag{16}$$

References

[1] Gary Schurman, *Integration By Parts - Weighted-Average Revenue Life,* January, 2020.

[2] Gary Schurman, *The Caculus of the Normal Distribution,* October, 2010.

[3] Gary Schurman, *The Poisson Distribution,* June, 2012.

[4] Gary Schurman, *Jump Diffusion - The Basic Affine Jump Diffusion Model,* March, 2021.